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$\Delta(54)$ flavor model for leptons and sleptons

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Abstract: We study a $\Delta(54)\times Z_2$ flavor model for leptons and sleptons. The tri-bimaximal mixing can be reproduced for arbitrary neutrino masses if certain vacuum alignments of scalar fields are realized. The deviation from the tri-bimaximal mixing of leptons is predicted. The predicted upper bound for $\sin\theta_{13}$ is 0.07. The value of $\sin\theta_{23}$ could be deviated from the maximal mixing considerably while $\sin\theta_{12}$ is hardly deviated from $1/\sqrt{3}$. We also study SUSY breaking terms in the slepton sector. Three families of left-handed and right-handed slepton masses are almost degenerate. Our model leads to smaller values of flavor changing neutral currents than the present experimental bounds.

Keywords: Discrete and Finite Symmetries, Supersymmetric Effective Theories

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Contents

1 Introduction

Recent experimental data of the neutrino oscillation indicate the tri-bimaximal form [1] of mixing angles in the lepton sector within a good accuracy [2, 3]. Thus, it is a promising step to study how to realize the tri-bimaximal mixing matrix, in order to understand the origin of the lepton flavor. Many authors have been attempting it by using various scenarios.

Non-Abelian discrete flavor symmetries are particularly well-known as one of quite verifiable methods to realize the tri-bimaximal mixing matrix. Non-Abelian discrete flavor symmetries can provide a natural guidance to constrain many free parameters in the Yukawa sector. Actually, several types of models with various non-Abelian discrete flavor symmetries have been proposed, such as S_3 [4]–[19], D_4 [20]–[24], D_6 [25], Q_4 [26], Q_6 [27], A_4 [28]–[50], T' [51]–[56], S_4 [57]–[65] and $\Delta(27)$ [66]–[70].

As another aspect, non-Abelian discrete flavor symmetries could also have an advantage of supersymmmetry (SUSY) flavor changing neutral currents (FCNCs).[1]

In general, there are a large number of free parameters mainly related to soft SUSY breaking terms even in the minimal supersymmetric standard model. However, once one could apply non-Abelian discrete flavor symmetries, those SUSY breaking parameters could be restricted and predictable in a way similar to the Yukawa sector. (See e.g. [24, 71–74].)

[1]While they might have a potential disadvantage of FCNC through extended Higgs fields, but one could avoid such a problem as far as one could stay at the scenarios with $SU(2)_L$ singlet extended Higgs fields.

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Thus, it would also be important to investigate non-Abelian discrete flavor symmetries from the viewpoint of the SUSY FCNCs.

In addition to the above (rather) bottom-up motivation, we also have a top-down motivation. Certain classes of non-Abelian flavor symmetries can be derived from superstring theories. For example, D_4 and $\Delta(54)$ flavor symmetries can be obtained in heterotic orbifold models [72, 75, 76]. In addition to these flavor symmetries, the $\Delta(27)$ flavor symmetry can be derived from magnetized/intersecting D-brane models [77]. Thus, it is quite important to study phenomenological aspects of these non-Abelian flavor symmetries.

Here, we focus on the $\Delta(54)$ discrete symmetry [78–80]. Although it includes several interesting aspects, few authors have considered up to now. The first aspect is that it consists of two types of Z_3 subgroups and an S_3 subgroup. The S_3 group is known as the minimal non-Abelian discrete symmetry, and the semi-direct product structure of $\Delta(54)$ between Z_3 and S_3 induces triplet irreducible representations. That suggests that the $\Delta(54)$ symmetry could lead to interesting models.

The authors have already presented a $\Delta(54)$ flavor model [80], in which the tri-bimaximal mixing of lepton flavors is reproduced in the vanishing limit of the solar neutrino mass-squared difference. Although the previous $\Delta(54)$ model is simple in the sense that the three generations of all lepton sectors are assigned to be $\Delta(54)$ triplets and it does not need additional symmetry such as Z_n, neutrino mass parameters must be tuned to reproduce experimental neutrino data by hand. In this paper, we present a new $\Delta(54)$ flavor model, which is improved to exactly provide the tri-bimaximal matrix for arbitrary neutrino masses. In the present model, the three generations of right-handed neutrinos are divided into singlet and doublet representations of $\Delta(54)$ and the additional Z_2 symmetry is imposed for the lepton sector.

This paper is organized as follows. In section 2, our new $\Delta(54) \times Z_2$ lepton flavor model is presented. In section 3 the possible deviation from the tri-bimaximal mixing is discussed, and in section 4 numerical results are presented. In section 5, soft SUSY breaking terms of sleptons are studied by taking account of FCNC constraints. Section 6 is devoted to summary and discussion. In appendix A, the analytic derivation of the mixing angles is given, and in appendix B, soft SUSY breaking terms in the previous $\Delta(54)$ flavor model [80] are summarized.

2 $\Delta(54)$ flavor model for leptons

In this section, we present the lepton flavor model with the $\Delta(54)$ flavor symmetry. We propose a new model within the framework of supersymmetric theory. Therefore, we can discuss this flavor symmetry in the slepton sector by constraining parameters of this model in the lepton sector.

The $\Delta(54)$ group is one of $\Delta(6n^2)$ series that has been discussed by a few authors [78, 79]. The group $\Delta(54)$ has irreducible representations 1_1, 1_2, 2_1, 2_2, 2_3, 2_4, $3_1^{(1)}$, $3_1^{(2)}$, $3_2^{(1)}$, and $3_2^{(2)}$. There are four triplets and products of $3_1^{(1)} \times 3_1^{(2)}$ and $3_2^{(1)} \times 3_2^{(2)}$ lead to the trivial singlet. The relevant multiplication rules are shown, e.g. in ref. [78, 79].

	(l_e, l_μ, l_τ)	(e^c, μ^c, τ^c)	N_e	(N_μ, N_τ)	$h_{u(d)}$	χ_1	(χ_2, χ_3)	(χ_4, χ_5)	(χ_6, χ_7, χ_8)
$\Delta(54)$	$3_1^{(1)}$	$3_2^{(2)}$	1_1	2_1	1_1	1_2	2_1	2_1	$3_1^{(2)}$
Z_2	1	-1	1	1	1	-1	-1	1	1

Table 1. Assignments of $\Delta(54) \times Z_2$ representations

Here, we present our model of the lepton flavor with the $\Delta(54)$ group. The triplet representations of the $\Delta(54)$ group correspond to the three generations of left-handed leptons and right-handed charged leptons while right-handed neutrinos are assigned to a singlet and a doublet of $\Delta(54)$. The left-handed leptons (l_e, l_μ, l_τ), the right-handed charged leptons (e^c, μ^c, τ^c) are assigned to be $3_1^{(1)}$ and $3_2^{(2)}$, respectively. For right-handed neutrinos, N_e^c is assigned to be 1_1 and (N_μ^c, N_τ^c) are assigned to be 2_1. Charged leptons cannot have mass terms unless new scalars χ_i are introduced in addition to the usual Higgs doublets, h_u and h_d. These new scalars are supposed to be $\mathrm{SU}(2)_L$ gauge singlets with vanishing $\mathrm{U}(1)_Y$ charge. Gauge singlets χ_1, (χ_2, χ_3), (χ_4, χ_5) and (χ_6, χ_7, χ_8) are assigned to be 1_2, 2_1, 2_1, and $3_1^{(2)}$, respectively. We also introduce Z_2 symmetry and the non-trivial charge is assigned to (e^c, μ^c, τ^c), χ_1 and (χ_2, χ_3). The particle assignments of $\Delta(54)$ and Z_2 are summarized in table 1. The usual Higgs doublets h_u and h_d are assigned to the trivial singlet 1_1. Here, all fields denote superfields, and in section 5 the superfield and its lowest scalar component are denoted by the same letter as a convention.

In this particle assignment, we consider the superpotential of leptons at the leading order in terms of the cut-off scale Λ, which is taken to be the Planck scale. For charged leptons, the superpotential of the Yukawa sector respecting $\Delta(54)$ and Z_2 symmetries is given by

$$\begin{aligned} W^{(l)} &= y_1^l (e^c l_e + \mu^c l_\mu + \tau^c l_\tau)\chi_1 h_d \\ &\quad + y_2^l \left[(-\omega e^c l_e - \omega^2 \mu^c l_\mu - \tau^c l_\tau)\chi_2 + (e^c l_e + \omega^2 \mu^c l_\mu + \omega \tau^c l_\tau)\chi_3\right] h_d/\Lambda, \end{aligned} \tag{2.1}$$

where $\omega = e^{2\pi i/3}$. For the right-handed Majorana neutrinos, we can write the superpotential as follows:

$$\begin{aligned} W^{(N)} &= M_1 N_e^c N_e^c + M_2 (N_\mu^c N_\tau^c + N_\tau^c N_\mu^c) \\ &\quad + y^N (N_\mu^c N_\mu^c \chi_4 + N_\tau^c N_\tau^c \chi_5). \end{aligned} \tag{2.2}$$

The superpotential for the Dirac neutrinos is given in leading order as

$$\begin{aligned} W^{(D)} &= y_1^D N_e^c (l_e \chi_6 + l_\mu \chi_7 + l_\tau \chi_8) h_u/\Lambda \\ &\quad + y_2^D \left[N_\mu^c (\omega l_e \chi_6 + \omega^2 l_\mu \chi_7 + l_\tau \chi_8) + N_\tau^c (l_e \chi_6 + \omega^2 l_\mu \chi_7 + \omega l_\tau \chi_8))\right] h_u/\Lambda. \end{aligned} \tag{2.3}$$

We assume that the scalar fields, $h_{u,d}$ and χ_i, develop their vacuum expectation values (VEVs) as follows:

$$\begin{aligned} \langle h_{u(d)} \rangle = v_{u(d)},\ \langle \chi_1 \rangle = u_1,\ \langle (\chi_2, \chi_3) \rangle = (u_2, u_3),\ \langle (\chi_4, \chi_5) \rangle = (u_4, u_5), \\ \langle (\chi_6, \chi_7, \chi_8) \rangle = (u_6, u_7, u_8). \end{aligned} \tag{2.4}$$

Then, the charged lepton mass matrix is diagonal:

$$M_l = y_1^l v_d \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_1 \end{pmatrix} + y_2^l v_d \begin{pmatrix} \omega\alpha_2 - \alpha_3 & 0 & 0 \\ 0 & \omega^2\alpha_2 - \omega^2\alpha_3 & 0 \\ 0 & 0 & \alpha_2 - \omega\alpha_3 \end{pmatrix}. \tag{2.5}$$

The right-handed Majorana mass matrix is given as

$$M_N = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & y^N\alpha_4\Lambda & M_2 \\ 0 & M_2 & y^N\alpha_5\Lambda \end{pmatrix}, \tag{2.6}$$

and the Dirac mass matrix of neutrinos is

$$M_D = y_1^D v_u \begin{pmatrix} \alpha_6 & \alpha_7 & \alpha_8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ \omega\alpha_6 & \omega^2\alpha_7 & \alpha_8 \\ \alpha_6 & \omega^2\alpha_7 & \omega\alpha_8 \end{pmatrix}, \tag{2.7}$$

where we denote $\alpha_i = u_i/\Lambda$ $(i = 1, \dots, 8)$. By using the seesaw mechanism $M_\nu = M_D^T M_N^{-1} M_D$, the neutrino mass matrix can be derived.

At first, we analyze the charged lepton sector. Masses are expressed by

$$\begin{pmatrix} m_e \\ m_\mu \\ m_\tau \end{pmatrix} = v_d \begin{pmatrix} 1 & \omega & -1 \\ 1 & \omega^2 & -\omega^2 \\ 1 & 1 & -\omega \end{pmatrix} \begin{pmatrix} y_1^l\alpha_1 \\ y_2^l\alpha_2 \\ y_2^l\alpha_3 \end{pmatrix}. \tag{2.8}$$

In order to estimate magnitudes of α_1, α_2 and α_3, we rewrite them as

$$\begin{pmatrix} y_1^l\alpha_1 \\ y_2^l\alpha_2 \\ y_2^l\alpha_3 \end{pmatrix} = \frac{1}{3v_d} \begin{pmatrix} 1 & 1 & 1 \\ -\omega-1 & \omega & 1 \\ -1 & -\omega & \omega+1 \end{pmatrix} \begin{pmatrix} m_e \\ m_\mu \\ m_\tau \end{pmatrix}, \tag{2.9}$$

which gives the relation of $|y_2^l\alpha_2| = |y_2^l\alpha_3|$. Inserting the experimental values of the charged lepton masses with $v_d \simeq 55\text{GeV}$ (i.e. $\tan\beta = 3$), we obtain numerical results

$$\begin{pmatrix} y_1^l\alpha_1 \\ y_2^l\alpha_2 \\ y_2^l\alpha_3 \end{pmatrix} = \begin{pmatrix} 1.14 \times 10^{-2} \\ 1.05 \times 10^{-2} e^{0.016i\pi} \\ 1.05 \times 10^{-2} e^{0.32i\pi} \end{pmatrix}. \tag{2.10}$$

Thus, it is found that $\alpha_i (i = 1, 2, 3)$ are of $\mathcal{O}(10^{-2})$ when Yukawa couplings are of $\mathcal{O}(1)$.

In our model, the lepton flavor mixing is originated from the structure of the neutrino mass matrix. To realize the tri-bimaximal mixing, we take

$$\alpha_5 = \omega\alpha_4\,, \qquad \alpha_6 = \alpha_7 = \alpha_8. \tag{2.11}$$

Now we have

$$\begin{aligned} M_\nu = &\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} y_1^D & 0 & 0 \\ 0 & y_2^D\omega & 0 \\ 0 & 0 & y_2^D \end{pmatrix} \begin{pmatrix} \frac{1}{M_1} & 0 & 0 \\ 0 & \frac{y^N\omega\alpha_4\Lambda}{(y^N\alpha_4\Lambda)^2\omega - M_2^2} & \frac{-M_2}{(y^N\alpha_4\Lambda)^2\omega - M_2^2} \\ 0 & \frac{-M_2}{(y^N\alpha_4\Lambda)^2\omega - M_2^2} & \frac{y^N\alpha_4\Lambda}{(y^N\alpha_4\Lambda)^2\omega - M_2^2} \end{pmatrix} \\ &\times \begin{pmatrix} y_1^D & 0 & 0 \\ 0 & y_2^D\omega & 0 \\ 0 & 0 & y_2^D \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \alpha_6^2 v_u^2. \end{aligned} \tag{2.12}$$

It can be rewritten as

$$M_\nu = 3c\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + (a-b-c)\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + 3b\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}, \tag{2.13}$$

where

$$a = \frac{(y_1^D)^2}{M_1}\alpha_6^2 v_u^2, \quad b = \frac{y^N (y_2^D)^2 \alpha_4 \Lambda}{(y^N\alpha_4\Lambda)^2\omega - M_2^2}\alpha_6^2 v_u^2, \quad c = \frac{-(y_2^D)^2\omega M2}{(y^N\alpha_4\Lambda)^2\omega - M_2^2}\alpha_6^2 v_u^2. \tag{2.14}$$

As well known, the neutrino mass matrix with the tri-bimaximal mixing is expressed in terms of neutrino mass eigenvalues m_1, m_2 and m_3 by

$$M_\nu = \frac{m_1+m_3}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}. \tag{2.15}$$

Therefore, our neutrino mass matrix M_ν gives the tri-bimaximal mixing matrix $U_{\rm tri}$ and mass eigenvalues as follows:

$$U_{\rm tri} = \begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}, \qquad m_1 = 3(b+c), \quad m_2 = 3a, \quad m_3 = 3(c-b)\,. \tag{2.16}$$

To compare with experimental values, we reparameterize $a = |a|$, $b = |b|e^{i\phi}$, $c = |c|e^{i\theta}$, then neutrino masses become

$$\begin{aligned}|m_1| =& 3\sqrt{|b|^2+|c|^2+2|b||c|\cos(\phi-\theta)},\\ |m_2| =& 3|a|,\\ |m_3| =& 3\sqrt{|b|^2+|c|^2-2|b||c|\cos(\phi-\theta)}.\end{aligned} \tag{2.17}$$

Mass-squared differences are given as

$$\begin{aligned}|m_3|^2-|m_1|^2 &= -36|b||c|\cos(\phi-\theta),\\ |m_2|^2-|m_1|^2 &= 9(|a|^2-|b|^2-|c|^2-2|b||c|\cos(\phi-\theta)).\end{aligned} \tag{2.18}$$

Considering normal-hierarchical neutrino masses, we take $|b| \simeq |c|$, $\phi - \theta \simeq \pi$, then, we get

$$m_1 \simeq 0, \quad m_2 \simeq 3|a|, \quad m_3 \simeq -6|b|e^{i\phi}. \tag{2.19}$$

Parameters a and b are estimated as $|a| \simeq \sqrt{\Delta m^2_{\rm sol}}/3$, $|b| \simeq \sqrt{\Delta m^2_{\rm atm}}/6$. which give the following relations by using of eq. (2.14):

$$\begin{aligned}\frac{(y_1^D)^2}{M_1}v_u^2\alpha_6^2 &\simeq \frac{\sqrt{\Delta m^2_{\rm sol}}}{3},\\ \frac{y^N(y_2^D)^2\alpha_4\Lambda}{(y^N\alpha_4\Lambda)^2\omega - M_2^2}v_u^2\alpha_6^2 \simeq \frac{(y_2^D)^2\omega M_2}{(y^N\alpha_4\Lambda)^2\omega - M_2^2}v_u^2\alpha_6^2 &\simeq \frac{\sqrt{\Delta m^2_{\rm atm}}}{6}e^{i\phi}.\end{aligned} \tag{2.20}$$

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Assuming α_4 and α_6 to be real, the Majorana phase of m_3 can be evaluated as

$$e^{i\phi} \simeq \frac{y^N (y_2^D)^2}{|y^N||y_2^D|^2} \frac{\sqrt{|y^N|^4 \alpha_4^4 \Lambda^4 + |M_2|^4 - (\omega (y^N)^2 M_2^{*2} + \omega^2 (y^{N*})^2 M_2{}^2) \alpha_4^2 \Lambda^2}}{\omega (y^N)^2 \alpha_4^2 \Lambda^2 - M_2^2}. \tag{2.21}$$

Because the second equation of (2.20) implies $y^N \alpha_4 \Lambda \simeq \omega M_2$, we set

$$y^N \alpha_4 \Lambda = (1+\epsilon)\omega M_2 \,, \tag{2.22}$$

where ϵ is tiny. By using the last equation in eq. (2.20), the atmospheric neutrino mass scale becomes

$$\sqrt{\Delta m_{\rm atm}^2} \simeq \frac{3|y^N||y_2^D|^2 \alpha_4 \Lambda v_u^2 \alpha_6^2}{\epsilon |M_2|^2}. \tag{2.23}$$

Now we can obtain magnitudes of α_4, α_6, and M_1 from experimental values, Yukawa couplings, cut-off scale Λ, Higgs VEVs, right-handed Majorana scale M_2, and small parameter ϵ. Concretely, let us take $v_u = 165\text{GeV}$, $\Lambda = 2.4 \times 10^{18}\text{GeV}$, $\Delta m_{\rm atm} = 2.4 \times 10^{-21}\text{GeV}^2$, $\Delta m_{\rm sol} = 8 \times 10^{-23}\text{GeV}^2$. Further, putting $|y_1^D| = |y_2^D| = |y^N| = 1$, $\epsilon = 10^{-2}$, and $M_2 = 10^{13}\text{GeV}$, we obtain typical values:

$$\begin{aligned} \alpha_4 &= \left| \frac{(1+\epsilon)\omega M_2}{y^N \Lambda} \right| \sim 4 \times 10^{-6}, \\ \alpha_6 &\sim \sqrt{\frac{\epsilon |M_2|^2 \sqrt{\Delta m_{\rm atm}^2}}{3|y^N||y_2^D|^2 \alpha_4 \Lambda v_u^2}} \sim 8 \times 10^{-3}, \\ M_1 &\sim \frac{3|y_1^D|^2 v_u^2 \alpha_6^2}{\sqrt{\Delta m_{\rm sol}^2}} \sim 5 \times 10^{11}\text{GeV}. \end{aligned} \tag{2.24}$$

In this way, we can estimate the magnitudes of α_4 and α_6, which are important parameters to calculate FCNC. Even if we consider the case of the inverted mass hierarchy, we easily find the almost same result of α_4 and α_6 as in eq. (2.24). In the section 4, we estimate them numerically by taking into account experimental data.

3 Deviation from tri-bimaximal mixing

The tri-bimaximal mixing can be exactly obtained under the condition of vacuum alignment in eq. (2.11). The mixing matrix is deviated from the tri-bimaximal matrix if the alignment of eq. (2.11) is shifted.

First, we discuss the effect of the deviation from $\alpha_5 = \omega \alpha_4$. To estimate this effect, we introduce a parameter δ with $\alpha_5 = \omega(1+\delta)\alpha_4$. The neutrino mass matrix is written as

$$M_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & (1+\delta)b & c \\ 0 & c & b \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{3.1}$$

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which is rewritten as

$$M_\nu = 3c\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + (a-b-c)\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + 3b\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + b\delta\begin{pmatrix}1&\omega&\omega^2\\\omega&\omega^2&1\\\omega^2&1&\omega\end{pmatrix}, \tag{3.2}$$

where the last matrix in the right hand side causes the deviation from the tri-bimaximal mixing. It can be diagonalized by the following mixing matrix

$$U = \begin{pmatrix} \frac{\sqrt{2}e^{-ip_1}}{\sqrt{3}}\cos\theta & \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}e^{-ip_1}}{\sqrt{3}}\sin\theta \\ -\frac{e^{-ip_1}\cos\theta+\sqrt{3}e^{-ip_2}\sin\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{e^{-ip_1}\sin\theta-\sqrt{3}e^{-ip_2}\cos\theta}{\sqrt{6}} \\ -\frac{e^{-ip_1}\cos\theta-\sqrt{3}e^{-ip_2}\sin\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{e^{-ip_1}\sin\theta+\sqrt{3}e^{-ip_2}\cos\theta}{\sqrt{6}} \end{pmatrix}, \tag{3.3}$$

where the phase difference $p_1 - p_2$ and additional mixing angle θ can be expressed by m_1, m_3, b, and δ as shown in appendix A. Then the lepton mixing matrix element U_{e3} can be estimated from these parameters. On the other hand, the element U_{e2} does not shift from $1/\sqrt{3}$. Numerical results are discussed in the next section.

We also consider the deviation from the alignment $\alpha_6 = \alpha_7 = \alpha_8$. New small parameters δ_1 and δ_2 are added to be $\alpha_7 = (1+\delta_1)\alpha_6$ and $\alpha_8 = (1+\delta_2)\alpha_6$. Then, we obtain

$$M_\nu = \begin{pmatrix}1&0&0\\0&1+\delta_1&0\\0&0&1+\delta_2\end{pmatrix} U_{\rm tri} \begin{pmatrix}m_1&0&0\\0&m_2&0\\0&0&m_3\end{pmatrix} U_{\rm tri}^\dagger \begin{pmatrix}1&0&0\\0&1+\delta_1&0\\0&0&1+\delta_2\end{pmatrix}. \tag{3.4}$$

It can be diagonalized by

$$U \simeq U_{\rm tri}\begin{pmatrix}\cos\theta_{12}&\sin\theta_{12}&0\\-\sin\theta_{12}&\cos\theta_{12}&0\\0&0&1\end{pmatrix}\begin{pmatrix}\cos\theta_{13}&0&\sin\theta_{13}\\0&1&0\\-\sin\theta_{13}&0&\cos\theta_{13}\end{pmatrix}\begin{pmatrix}1&0&0\\0&\cos\theta_{23}&\sin\theta_{23}\\0&-\sin\theta_{23}&\cos\theta_{23}\end{pmatrix}, \tag{3.5}$$

where

$$\begin{aligned} \theta_{12} &\simeq -\frac{m_1+m_2}{3\sqrt{2}(m_2-m_1)}(\delta_1+\delta_2), \quad \theta_{13} \simeq \frac{m_1+m_3}{2\sqrt{3}(m_3-m_1)}(\delta_1-\delta_2), \\ \theta_{23} &\simeq -\frac{m_2+m_3}{\sqrt{6}(m_3-m_2)}(\delta_1-\delta_2). \end{aligned} \tag{3.6}$$

Supposing the normal hierarchy of neutrino masses $m_3 \gg m_2, m_1$, we find

$$\begin{aligned} U_{e3} &\simeq \frac{\sqrt{2}m_2}{3m_3}(\delta_2-\delta_1), \\ U_{\mu 2} &\simeq -\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{4}(\delta_2-\delta_1). \end{aligned} \tag{3.7}$$

Since U_{e3} is strongly suppressed by order δ_i and the ratio m_2/m_3, we consider no more the deviation of $\alpha_6 = \alpha_7 = \alpha_8$ in our numerically work.

4 Numerical results

In this section, we discuss the magnitude of the deviation from the tri-bimaximal matrix numerically. By restricting neutrino masses and mixing angles within experimental errors, magnitudes of α_4 and α_6 can be obtained as discussed in the section 3. We consider the case of the normal hierarchy of neutrino masses[2] as discussed in section 2.

Input data of masses and mixing angles are taken in the region of 3σ of the experimental data [2, 3]:

$$\begin{aligned}
&\Delta m^2_{\rm atm} = (2.07 \sim 2.75) \times 10^{-3}{\rm eV}^2\,, \qquad \Delta m^2_{\rm sol} = (7.05 \sim 8.34) \times 10^{-5}{\rm eV}^2\,,\\
&\sin^2\theta_{\rm atm} = 0.36 \sim 0.67\,, \qquad \sin^2\theta_{\rm sol} = 0.25 \sim 0.37\,, \quad \sin^2\theta_{\rm reactor} \le 0.056\;.
\end{aligned} \tag{4.1}$$

Yukawa couplings y_1^D and y_2^D are complex. Those absolute values and phases are chosen from -1 to 1 and 0 to 2π at random, respectively. On the other hand, y^N is given in eq. (2.22). We search the experimentally allowed region by diagonalizing the neutrino mass matrix with varying the parameters α_4, α_6, M_2, and ϵ in eq. (2.22), which are taken to be real.

As discussed in section 3, we take $\alpha_5 = \omega\alpha_4(1+\delta)$, where δ is a complex parameter, while we take $\alpha_6 = \alpha_7 = \alpha_8$. By varying δ, the mixing matrix is deviated from the tri-bimaximal matrix. The neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & b(1+\delta) & c \\ 0 & c & b \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{4.2}$$

We present the numerical result in figure 1. In figure 1(a), we show the allowed region on the plane of $\sin^2\theta_{23} - \sin^2\theta_{12}$. The mixing parameter $\sin\theta_{12}$ is hardly deviated from the tri-maximal mixing $1/\sqrt{3}$ as expected from eq. (3.3). On the other hand, $\sin\theta_{23}$ is deviated from the bi-maximal mixing considerably. We show allowed values of $\sin^2\theta_{13}$ versus $\sin^2\theta_{23}$ in figure 1(b). The predicted upper bound of $\sin^2\theta_{13}$ is 5×10^{-3}. As seen in figure 1(c), $\sin\theta_{13}$ is proportional to the magnitude $|\delta|$, which is bounded by 0.3 due to the experimental data of the neutrino mass-squared differences. In figure 1(d), we show the allowed region on the $\alpha_4 - \alpha_6$ plane. Since α_6 larger than 10^{-2} is dangerous for the FCNC constraints as discussed in the next section, we have searched the parameter space in $\alpha_6 \le 10^{-2}$. Then we find $\alpha_4 = 10^{-6} \sim 10^{-4}$. In these numerical calculations, we take $10^9{\rm GeV} < M_2 < 10^{16}{\rm GeV}$ and $\epsilon = 10^{-3} \sim 10^{-1}$.

5 SUSY breaking terms

In this section, we study SUSY breaking terms, i.e., sfermion masses and scalar trilinear couplings, which are predicted in our $\Delta(54)\times Z_2$ model. We consider the gravity mediation within the framework of supergravity theory. We assume that non-vanishing F-terms

[2]We have not presented the case of the inverted hierarchy of neutrino masses since numerical results are almost same as ones in the case of the normal hierarchy.

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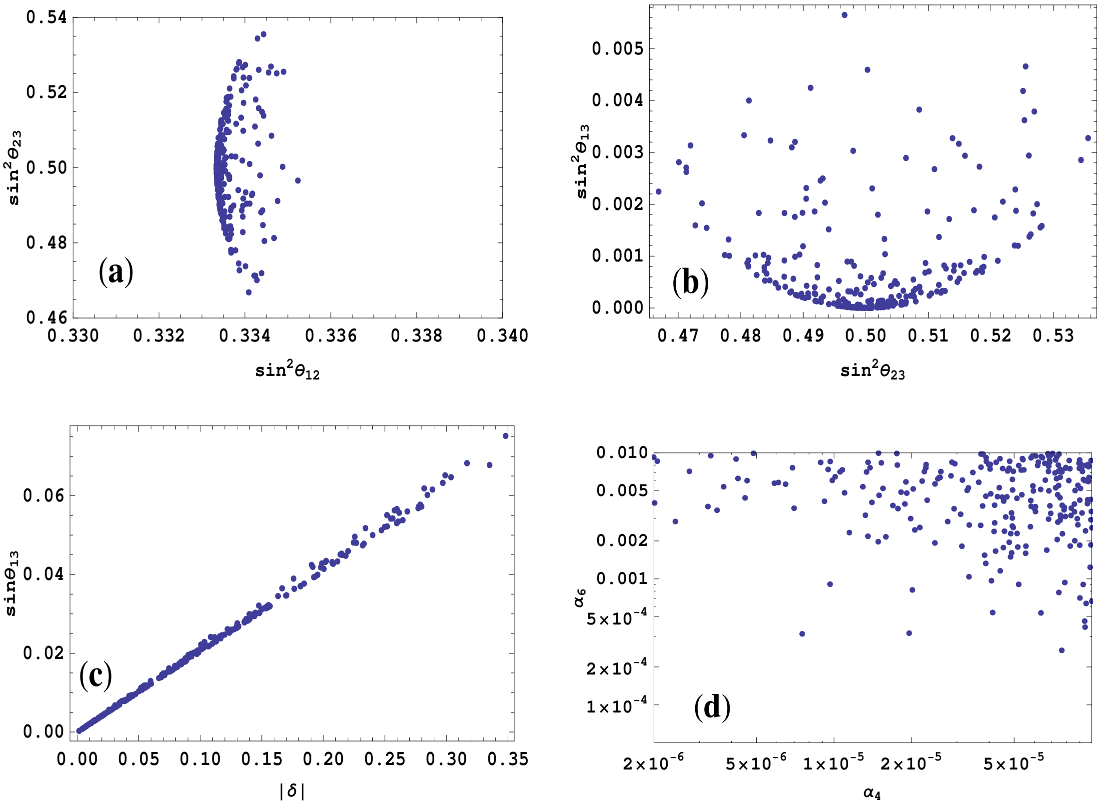

Figure 1. Predicted plots on the plane of (a) $\sin^2\theta_{23} - \sin^2\theta_{12}$, (b) $\sin^2\theta_{23} - \sin^2\theta_{13}$, (c)$|\delta| - \sin\theta_{13}$ and (d) $\alpha_4 - \alpha_6$ in the region of $\alpha_6 \le 10^{-2}$.

of gauge and flavor singlet (moduli) fields Z and gauge singlet fields χ_i $(i = 1, \ldots, 8)$ contribute to SUSY breaking. Their F-components are written as

$$F^{\Phi_k} = -e^{\frac{K}{2M_p^2}} K^{\Phi_k \bar{I}} \left(\partial_{\bar{I}} \bar{W} + \frac{K_{\bar{I}}}{M_p^2} \bar{W} \right), \tag{5.1}$$

where K denotes the Kähler potential, $K_{\bar{I}J}$ denotes second derivatives by fields, i.e. $K_{\bar{I}J} = \partial_{\bar{I}} \partial_J K$ and $K^{\bar{I}J}$ is its inverse. Here the fields Φ_k correspond to the moduli fields Z and gauge singlet fields χ_i $(i = 1, \ldots, 8)$. The VEVs of F_{Φ_k}/Φ_k are estimated as $\langle F_{\Phi_k}/\Phi_k \rangle = \mathcal{O}(m_{3/2})$, where $m_{3/2}$ denotes the gravitino mass, which is obtained as $m_{3/2} = \langle e^{K/2M_p^2} W/M_p^2 \rangle$.

5.1 Slepton mass matrices

First, let us study soft scalar masses. Within the framework of supergravity theory, soft scalar mass squared is obtained as [83]

$$m^2_{\bar{I}J} K_{\bar{I}J} = m^2_{3/2} K_{\bar{I}J} + |F^{\Phi_k}|^2 \partial_{\Phi_k} \partial_{\bar{\Phi}_k} K_{\bar{I}J} - |F^{\Phi_k}|^2 \partial_{\bar{\Phi}_k} K_{\bar{I}L} \partial_{\Phi_k} K_{\bar{M}J} K^{L\bar{M}}. \tag{5.2}$$

The invariance under the $\Delta(54) \times Z_2$ flavor symmetry as well as the gauge invariance requires the following form of the Kähler potential of l_I and e_I $(I = e, \mu, \tau)$

$$K = Z^{(L)}(Z) \sum_{I=e,\mu,\tau} |l_I|^2 + Z^{(R)}(Z) \sum_{I=e,\mu,\tau} |e_I|^2, \tag{5.3}$$

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at the lowest level, where $Z^{(L)}(Z)$ and $Z^{(R)}(Z)$ are arbitrary functions of the singlet fields Z. By use of the formula (5.2) with the Kähler potential (5.3), we obtain the following matrix form of soft scalar masses squared for left-handed and right-handed charged sleptons,

$$(m^2_{\tilde{L}})_{IJ} = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix}, \qquad (m^2_{\tilde{R}})_{IJ} = \begin{pmatrix} m_R^2 & 0 & 0 \\ 0 & m_R^2 & 0 \\ 0 & 0 & m_R^2 \end{pmatrix}. \tag{5.4}$$

That is, both matrices are proportional to the (3×3) identity matrix. This form would be obvious because (l_e, l_μ, l_τ) and (e^c, μ^c, τ^c) are $\Delta(54)$ triplets. At any rate, it is the prediction of our model that three families of left-handed and right-handed masses are degenerate.

The above prediction holds exactly before $\Delta(54) \times Z_2$ is broken, but its breaking would change the form. Next, we study effects due to $\Delta(54) \times Z_2$ breaking by χ_i. That is, we estimate corrections to the Kähler potential including χ_i. The VEVs of $\chi_{4,5}$ are much smaller than the others. Thus, we concentrate on corrections including χ_i with $i = 1, 2, 3, 6, 7, 8$.

In our model, the left-handed charged leptons (l_e, l_μ, l_τ) are assigned to be $3_1^{(1)}$ and its conjugate representation is $3_1^{(2)}$. Their multiplication rule is written as

$$3_1^{(1)} \times 3_1^{(2)} = 1_1 + 2_1 + 2_2 + 2_3 + 2_4, \tag{5.5}$$

and that is written more explicitly in terms of elements as

$$\begin{aligned}(x_1, x_2, x_3)_{3_1^{(1)}} \times (y_1, y_2, y_3)_{3_1^{(2)}} = & (x_1 y_1 + x_2 y_2 + x_3 y_3)_{1_1} \\ & + (x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3, \omega x_1 y_1 + \omega^2 x_2 y_2 + x_3 y_3)_{2_1} \\ & + (x_1 y_2 + \omega^2 x_2 y_3 + \omega x_3 y_1, \omega x_1 y_3 + \omega^2 x_2 y_1 + x_3 y_2)_{2_2} \\ & + (x_1 y_3 + \omega^2 x_2 y_1 + \omega x_3 y_2, \omega x_1 y_2 + \omega^2 x_2 y_3 + x_3 y_1)_{2_3} \\ & + (x_1 y_3 + x_2 y_1 + x_3 y_2, x_1 y_2 + x_2 y_3 + x_3 y_1)_{2_4}. \end{aligned} \tag{5.6}$$

By use of this multiplication rule, we can find that linear terms of χ_i for $i = 1, 2, 3, 6, 7, 8$ do not appear in corrections of the Kähler potential (5.3). Although linear terms of $\chi_{4,5}$ can appear in diagonal elements of Kähler metric, those corrections are not important as said above. Thus, let us estimate corrections including $\chi_i \chi_k$ as well as $\chi_i \chi_k^*$ for $i, k = 1, 2, 3, 6, 7, 8$. The $\Delta(54) \times Z_2$ flavor symmetric invariance allows only the terms such as $\chi_i \chi_k^*$ for $i, k = 6, 7, 8$ to appear in off-diagonal entries of the Kähler metric of (l_e, l_μ, l_τ). For example, the (1,2) entry of the Kähler metric would have correction terms like e.g.

$$\Delta K = \frac{K'(Z)}{\Lambda^2} \chi_i \chi_k^* l_1 l_2^* + \cdots, \tag{5.7}$$

where $K'(Z)$ is an arbitrary function of Z. On the other hand, the terms such as $\chi_i \chi_k^*$ for $i, k = 1, 2, 3$ can appear in the diagonal entries, but such corrections only change the overall factor of the form Kähler potential (5.3). When we take into account the corrections from $\chi_i \chi_k^*$ for $i, k = 6, 7, 8$ to the Kähler potential, the soft scalar masses squared for left-handed charged sleptons have the following corrections,

$$(m^2_{\tilde{L}})_{IJ} = m_L^2 \begin{pmatrix} 1 + \mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) \\ \mathcal{O}(\alpha_6^2) & 1 + \mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) \\ \mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) & 1 + \mathcal{O}(\alpha_6^2) \end{pmatrix}. \tag{5.8}$$

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Similarly, when we include the same level of corrections, the soft scalar masses squared for right-handed sleptons are obtained as

$$(m^2_{\tilde{R}})_{IJ} = m^2_R \begin{pmatrix} 1+\mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) \\ \mathcal{O}(\alpha_6^2) & 1+\mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) \\ \mathcal{O}(\alpha_6^2) & \mathcal{O}(\alpha_6^2) & 1+\mathcal{O}(\alpha_6^2) \end{pmatrix}. \tag{5.9}$$

These deviations may not be important for direct measurement of slepton masses. However, the off-diagonal entries in the SCKM basis[3] are constrained by the FCNC experiments [84]. Our model predicts

$$(\Delta_{LL})_{12} = \frac{(m^2_{\tilde{L}})^{(SCKM)}_{12}}{(m^2_{\tilde{L}})_{11}} = \mathcal{O}(\alpha_6^2), \qquad (\Delta_{RR})_{12} = \frac{(m^2_{\tilde{R}})^{(SCKM)}_{12}}{(m^2_{\tilde{R}})_{11}} = \mathcal{O}(\alpha_6^2). \tag{5.10}$$

Recall that the diagonalizing matrices of left-handed and right-handed fermions are almost the identity matrix. The $\mu \to e\gamma$ experiments constrain these values as $(\Delta_{LL,RR})_{12} \leq \mathcal{O}(10^{-3})$ [84], when $m_{L,R} = 100\,\mathrm{GeV}$. On the other hand, the parameter space in the previous section corresponds to $\alpha_6 \leq 10^{-2}$ and leads to $(\Delta_{LL,RR})_{12} \leq \mathcal{O}(10^{-4})$. Thus, our parameter region would be favorable also from the viewpoint of the FCNC constraints.

5.2 A-terms

Here, let us study scalar trilinear couplings, i.e. the so-called A-terms. The A-terms among left-handed and right-handed sleptons and Higgs scalar fields are obtained in the gravity mediation as [83]

$$h_{IJ} l_J e_I H_d = h^{(Y)}_{IJ} l_J e_I H_d + h^{(K)}_{IJ} l_J e_I H_d, \tag{5.11}$$

where

$$\begin{aligned} h^{(Y)}_{IJ} &= F^{\Phi_k} \langle \partial_{\Phi_k} \tilde{y}_{IJ} \rangle, \\ h^{(K)}_{IJ} l_J e_I H_d &= -\langle \tilde{y}_{LJ} \rangle l_J e_I H_d F^{\Phi_k} K^{L\bar{L}} \partial_{\Phi_k} K_{\bar{L}I} \\ &\quad -\langle \tilde{y}_{IM} \rangle l_J e_I H_d F^{\Phi_k} K^{M\bar{M}} \partial_{\Phi_k} K_{\bar{M}J} \\ &\quad -\langle \tilde{y}_{IJ} \rangle l_J e_I H_d F^{\Phi_k} K^{H_d} \partial_{\Phi_k} K_{H_d}, \end{aligned} \tag{5.12}$$

where K_{H_d} denotes the Kähler metric of H_d. In addition, $\tilde{y}_{IJ}$ denotes effective Yukawa couplings and in our model it corresponds to

$$\tilde{y}_{IJ} = y^l_1 \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_1 & 0 \\ 0 & 0 & \alpha_1 \end{pmatrix} + y^l_2 \begin{pmatrix} \omega\alpha_2 - \alpha_3 & 0 & 0 \\ 0 & \omega^2\alpha_2 - \omega^2\alpha_3 & 0 \\ 0 & 0 & \alpha_2 - \omega\alpha_3 \end{pmatrix}. \tag{5.13}$$

Then, we obtain

$$h^{(Y)}_{IJ} = y^l_1 \begin{pmatrix} \tilde{F}^{\alpha_1} & 0 & 0 \\ 0 & \tilde{F}^{\alpha_1} & 0 \\ 0 & 0 & \tilde{F}^{\alpha_1} \end{pmatrix} + y^l_2 \begin{pmatrix} \omega\tilde{F}^{\alpha_2} - \tilde{F}^{\alpha_3} & 0 & 0 \\ 0 & \omega^2\tilde{F}^{\alpha_2} - \omega^2\tilde{F}^{\alpha_3} & 0 \\ 0 & 0 & \tilde{F}^{\alpha_2} - \omega\tilde{F}^{\alpha_3} \end{pmatrix}, \tag{5.14}$$

[3] The SCKM basis is the basis, where fermion mass matrix is diagonal.

where $\tilde{F}^{\alpha_i} = F^{\alpha_i}/\alpha_i$. Because of $\tilde{F}^{\alpha_i} = \mathcal{O}(m_{3/2})$, all the diagonal entries of $h^{(Y)}_{IJ}$ may be of $\mathcal{O}(y^l_1 m_{3/2})$. That would cause a problem. If $|h_{IJ}/\tilde{y}_{IJ}|$ is large compared with slepton masses, there would be a minimum, where charge is broken [85].[4]

To avoid this, we require that $F^{\alpha_1}/\alpha_1 = F^{\alpha_2}/\alpha_2 = F^{\alpha_3}/\alpha_3$. Such a relation can be realized if the Kähler metric of χ_i for $i = 1, 2, 3$ is the same and the non-perturbative superpotential leading to SUSY breaking does not include $\chi_{1,2,3}$. In this case, we obtain $F^{\alpha_i}/\alpha_i = m_{3/2}$ for $i = 1, 2, 3$. Then, we obtain

$$h^{(Y)}_{IJ} = \tilde{y}_{IJ} m_{3/2}, \tag{5.15}$$

that is, $h^{(Y)}_{11} = \mathcal{O}(m_{3/2} m_e/m_\tau)$ and $h^{(Y)}_{22} = \mathcal{O}(m_{3/2} m_\mu/m_\tau)$.

Next, we estimate $h^{(K)}_{IJ}$. When we neglect correction terms and use the lowest level of Kähler potential (5.3), we obtain

$$h^{(K)}_{IJ} = \tilde{y}_{IJ} A_0, \tag{5.16}$$

where $A_0 = \mathcal{O}(m_{3/2})$. Furthermore, we take into account corrections to the Kähler potential including χ_i, and we obtain

$$h^{(K)}_{IJ} v_d = \begin{pmatrix} m_e A_0 + \mathcal{O}(m_e \alpha_6^2 m_{3/2}) & \mathcal{O}(m_\mu \alpha_6^2 m_{3/2}) & \mathcal{O}(m_\tau \alpha_6^2 m_{3/2}) \\ \mathcal{O}(m_\mu \alpha_6^2 m_{3/2}) & m_\mu A_0 + \mathcal{O}(m_\mu \alpha_6^2 m_{3/2}) & \mathcal{O}(m_\tau \alpha_6^2 m_{3/2} \\ \mathcal{O}(m_\tau \alpha_6^2 m_{3/2} & \mathcal{O}(m_\tau \alpha_6^2 m_{3/2} & m_\tau A_0 + \mathcal{O}(m_\tau \alpha_6^2 m^2_{3/2}) \end{pmatrix}. \tag{5.17}$$

This structure does not change except replacing A_0 by $A_0 + m_{3/2}$ when we include $h^{(Y)}_{IJ}$ (5.15). Then, our model predicts $h_{12} v_d / m^2_{3/2} = \mathcal{O}(\alpha_6^2 m_\mu / m_{3/2})$. This ratio is constrained less than $\mathcal{O}(10^{-6})$ by the $\mu \to e\gamma$ experiments when the slepton mass is equal to 100 GeV. That is, the parameter region with $\alpha_6^2 \leq \mathcal{O}(10^{-3})$ is favorable. Thus, our parameter region $\alpha_6 \leq 10^{-2}$ in the previous section is favorable again from the FCNC constraints on the A-terms.

6 Summary and discussion

We have presented the flavor model for the lepton mass matrices by using the discrete symmetry $\Delta(54)$, which could be originated from heterotic string orbifold models or magnetized/intersecting D-brane models. The left-handed leptons, the right-handed charged leptons and the right-handed neutrinos are assigned to be $3^{(1)}_1$, $3^{(2)}_2$ and $1_1 + 2_1$, respectively. We introduce gauge singlets χ_1, (χ_2, χ_3), (χ_4, χ_5), and (χ_6, χ_7, χ_8), which are assigned to be 1_2, 2_1, 2_1, and $3^{(2)}_1$ of the $\Delta(54)$ representations, respectively.

The discrete symmetry reduces fine tuning to get the tri-bimaximal mixing for arbitrary neutrino masses. However, some fine tuning is implicitly introduced in vacuum alignments of scalar fields if those are not guaranteed in our model. Therefore, we should discuss the origin of vacuum alignments. One way is to analyze the scalar potential. Unfortunately, the scalar potential is very complicated in our model since there are nine

[4] If a decay rate from the realistic minimum to such charge breaking minimum is sufficiently small compared with the age of the universe, that might not be a problem.

scalar fields which develop their VEVs. We can only say that our desired VEVs are just one of solutions to realize the potential minimum. We can also discuss new methods [81, 82] in the extra dimensional theory, which may naturally lead desired vacuum alignments. Details will be studied elsewhere.

In our model, we predict the upper bound 0.07 for $\sin\theta_{13}$. The magnitudes of $\sin\theta_{23}$ could be deviated from the bi-maximal mixing considerably, but $\sin\theta_{12}$ is hardly deviated from $1/\sqrt{3}$.

We have also studied SUSY breaking terms. It is the prediction of our flavor model that three families of left-handed and right-handed slepton masses are almost degenerate. Our model leads to smaller values of FCNCs than the present experimental bounds.

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A Derivation mixing angles from mass matrix

We show analytic expressions for the mixing matrix. By the unitary transformation, the neutrino mass matrix M_ν in eq. (3.2) becomes

$$\tilde{M}_\nu = U_{\rm tri}^\dagger M_\nu U_{\rm tri} = \begin{pmatrix} m_1 + \frac{3}{2}b\delta & 0 & \frac{3\omega^2-3\omega}{2\sqrt{3}}b\delta \\ 0 & m_2 & 0 \\ \frac{3\omega^2-3\omega}{2\sqrt{3}}b\delta & 0 & m_3 - \frac{3}{2}b\delta \end{pmatrix}, \tag{A.1}$$

where

$$U_{\rm tri} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \tag{A.2}$$

Therefore, the deviation from the tri-bimaximal mixing can be expressed by diagonalizing 2×2 matrix. The matrix $\tilde{M}_\nu$ can be diagonalized by

$$\begin{pmatrix} m_1 + \frac{3}{2}b\delta & 0 & \frac{3\omega^2-3\omega}{2\sqrt{3}}b\delta \\ 0 & m_2 & 0 \\ \frac{3\omega^2-3\omega}{2\sqrt{3}}b\delta & 0 & m_3 - \frac{3}{2}b\delta \end{pmatrix} = P^{-1}V_{13} \begin{pmatrix} m_1' & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3' \end{pmatrix} V_{13}^T P^{-1}, \tag{A.3}$$

where

$$V_{13} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \quad P = \begin{pmatrix} e^{ip_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{ip_2} \end{pmatrix}. \tag{A.4}$$

We introduce new phase parameters as follows:

$$m_a = m_1 + \frac{3}{2}b\delta = |m_a|e^{i\mu_a}, \quad m_b = m_3 - \frac{3}{2}b\delta = |m_b|e^{i\mu_b}, \quad b = |b|e^{i\beta}, \quad \delta = |\delta|e^{i\xi}. \tag{A.5}$$

Then, we find the phase difference $p_1 - p_2$ and an additional mixing angle θ as

$$\begin{aligned}\tan(p_1-p_2) &= \frac{-|m_a|\sin(\mu_a-\pi/2-\beta-\xi)+|m_b|\sin(\mu_b-\pi/2-\beta-\xi)}{|m_a|\cos(\mu_a-\pi/2-\beta-\xi)+|m_b|\cos(\mu_b-\pi/2-\beta-\xi)},\\ \tan(2\theta) &= \frac{-3|b||\delta|}{|m_a|\cos(\mu_a-\pi/2-\beta-\xi+p_1-p_2)-|m_b|\cos(\mu_b-\pi/2-\beta-\xi-p_1+p_2)},\end{aligned}$$

and neutrino masses as

$$\begin{aligned}m_1' &= c^2|m_a|e^{i(\mu_a+2p_1)} + s^2|m_b|e^{i(\mu_b+2p_2)} - 3cs|b||\delta|e^{i(\pi/2+\beta+\xi+p_1+p_2)},\\ m_3' &= s^2|m_a|e^{i(\mu_a+2p_1)} + c^2|m_b|e^{i(\mu_b+2p_2)} + 3cs|b||\delta|e^{i(\pi/2+\beta+\xi+p_1+p_2)}.\end{aligned} \tag{A.6}$$

Mixing matrix of this situation can be expressed by

$$U = U_{\rm tri}P^{-1}V_{13} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}}e^{-ip_1}\cos\theta & \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}e^{-ip_1}}{\sqrt{3}}\sin\theta \\ -\frac{e^{-ip_1}\cos\theta+\sqrt{3}e^{-ip_2}\sin\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{e^{-ip_1}\sin\theta-\sqrt{3}e^{-ip_2}\cos\theta}{\sqrt{6}} \\ -\frac{e^{-ip_1}\cos\theta-\sqrt{3}e^{-ip_2}\sin\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{e^{-ip_1}\sin\theta+\sqrt{3}e^{-ip_2}\cos\theta}{\sqrt{6}} \end{pmatrix}. \tag{A.7}$$

In the same way, we diagonalize another neutrino mass matrix M_ν in eq. (3.4) as

$$\tilde{M}_\nu = U_{\rm tri}^\dagger M_\nu U_{\rm tri} = \begin{pmatrix} m_1 + \frac{\delta_1+\delta_2}{3}m_1 & -\frac{(\delta_1+\delta_2)(m_1+m_2)}{3\sqrt{2}} & \frac{(\delta_1-\delta_2)(m_1+m_3)}{2\sqrt{3}} \\ -\frac{(\delta_1+\delta_2)(m_1+m_2)}{3\sqrt{2}} & m_2 + \frac{2(\delta_1+\delta_2)}{3}m_2 & -\frac{(\delta_1-\delta_2)(m_2+m_3)}{\sqrt{6}} \\ \frac{(\delta_1-\delta_2)(m_1+m_3)}{2\sqrt{3}} & -\frac{(\delta_1-\delta_2)(m_2+m_3)}{\sqrt{6}} & m_3 + (\delta_1+\delta_2)m_3 \end{pmatrix}. \tag{A.8}$$

For simplicity, we assume that δ_1 and δ_2 are real and neglect δ_1^2 and δ_2^2, then new mass eigenvalues are approximately

$$m_1' \simeq \frac{3+\delta_1+\delta_2}{3}m_1, \quad m_2' \simeq \frac{3+2(\delta_1+\delta_2)}{3}m_2, \quad m_3' \simeq (1+\delta_1+\delta_2)m_3. \tag{A.9}$$

Similarly, mixing matrix to diagonalize $\tilde{M}_\nu$ can be also expressed in terms of δ_1 and δ_2 as

$$U = U_{\rm tri}\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \tag{A.10}$$

where

$$\begin{aligned}&\theta_{12} \simeq -\frac{m_1+m_2}{3\sqrt{2}(m_2-m_1)}(\delta_1+\delta_2), \quad \theta_{13} \simeq \frac{m_1+m_3}{2\sqrt{3}(m_3-m_1)}(\delta_1-\delta_2),\\ &\theta_{23} \simeq -\frac{m_2+m_3}{\sqrt{6}(m_3-m_2)}(\delta_1-\delta_2).\end{aligned} \tag{A.11}$$

	(L_e, L_μ, L_τ)	$(e^c_e, e^c_\mu, e^c_\tau)$	$(N^c_e, N^c_\mu, N^c_\tau)$	$h_{u(d)}$	χ_1	(χ_2, χ_3)	(χ_4, χ_5, χ_6)
$\Delta(54)$	$3_1^{(1)}$	$3_2^{(2)}$	$3_1^{(2)}$	1_1	1_2	2_1	$3_1^{(2)}$

Table 2. Assignments of $\Delta(54)$ representations

B Another $\Delta(54)$ flavor model

Here, for comparison, we study soft SUSY breaking terms derived from the $\Delta(54)$ flavor model, which was discussed in ref. [80]. In this model, the flavor symmetry is $\Delta(54)$, but there is no additional Z_2 flavor symmetry. We introduce gauge singlets, χ_i for $i = 1, \ldots, 6$, and assignments of $\Delta(54)$ representations are shown in table 2.

We assume that χ_i develop their VEVs and parameterize them as $\alpha_i = \chi_i/\Lambda$. To realize lepton masses and mixing angles, values of parameters are required as $\alpha_{1,2,3} = \mathcal{O}(10^{-2})$ and $\alpha_{4,5} = \mathcal{O}(10^{-4}) - \mathcal{O}(10^{-3})$. (See for details ref. [80].)

Now, let us study soft SUSY breaking scalar masses. Both the left-handed and right-handed leptons are $\Delta(54)$ triplets in this model, too. At the lowest order, we obtain the same Kähler potential for leptons as (5.3). Then, at this level, the prediction for slepton masses is the same as (5.4). That is, three families of left-handed and right-handed slepton masses are degenerate. Next, we consider the corrections including χ_i. Since $\alpha_{1,2,3}$ are larger than $\alpha_{4,5}$, the corrections including the form $\alpha_{1,2,3}$ are important. We examine which corrections including $\chi_{1,2,3}$ are allowed by the $\Delta(54)$ symmetry. Then, the resultant slepton masses squared have the following corrections in the SCKM basis,

$$(m^2_{\tilde{L}})_{ij} = m^2_L \begin{pmatrix} 1+\mathcal{O}(\alpha_1) & \mathcal{O}(\alpha_1^2) & \mathcal{O}(\alpha_1^2) \\ \mathcal{O}(\alpha_1^2) & 1+\mathcal{O}(\alpha_1) & \mathcal{O}(\alpha_1^2) \\ \mathcal{O}(\alpha_1^2) & \mathcal{O}(\alpha_1^2) & 1+\mathcal{O}(\alpha_1) \end{pmatrix}, \tag{B.1}$$

for the left-handed sleptons, and

$$(m^2_{\tilde{R}})_{ij} = m^2_R \begin{pmatrix} 1+\mathcal{O}(\alpha_1) & \mathcal{O}(\alpha_1^2) & \mathcal{O}(\alpha_1^2) \\ \mathcal{O}(\alpha_1^2) & 1+\mathcal{O}(\alpha_1) & \mathcal{O}(\alpha_1^2) \\ \mathcal{O}(\alpha_1^2) & \mathcal{O}(\alpha_1^2) & 1+\mathcal{O}(\alpha_1) \end{pmatrix}, \tag{B.2}$$

for the right-handed sleptons. Thus, we obtain $(\Delta_{LL})_{12} = (\Delta_{RR})_{12} = \mathcal{O}(\alpha_1^2)$. To realize the lepton masses, we need $\alpha = \mathcal{O}(10^{-2})$. Such a parameter region is also favorable from the FCNC constraint.

Similarly, we can estimate the A-terms. When we take into account important corrections, the A-term matrix is estimated as

$$h_{IJ}v_d = \begin{pmatrix} m_e A_0 + \mathcal{O}(m_e\tilde{\alpha}m_{3/2}) & \mathcal{O}(m_\mu\tilde{\alpha}^2 m_{3/2}) & \mathcal{O}(m_\tau\tilde{\alpha}^2 m_{3/2}) \\ \mathcal{O}(m_\mu\tilde{\alpha}^2 m_{3/2}) & m_\mu A_0 + \mathcal{O}(m_\mu\tilde{\alpha}m_{3/2}) & \mathcal{O}(m_\tau\tilde{\alpha}^2 m_{3/2} \\ \mathcal{O}(m_\tau\tilde{\alpha}^2 m_{3/2} & \mathcal{O}(m_\tau\tilde{\alpha}^2 m_{3/2} & m_\tau A_0 + \mathcal{O}(m_\tau\tilde{\alpha}m_{3/2}) \end{pmatrix}, \tag{B.3}$$

where $A_0 = \mathcal{O}(m_{3/2})$ and we have also assumed that $F^{\alpha_1}/\alpha_1 = F^{\alpha_2}/\alpha_2 = F^{\alpha_3}/\alpha_3$ as in section 5. When $m_{3/2} = 100\,\text{GeV}$, we obtain $h_{12}v_d/m^2_{3/2} = \mathcal{O}(10^{-7})$ for $\alpha = \mathcal{O}(10^{-2})$. Thus, the parameter region for α_i is favorable again from the FCNC constraint of A-terms.

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References

[1] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, Phys. Lett. **B 530** (2002) 167 [hep-ph/0202074] [SPIRES];
P.F. Harrison and W.G. Scott, *Symmetries and generalisations of tri-bimaximal neutrino mixing*, Phys. Lett. **B 535** (2002) 163 [hep-ph/0203209] [SPIRES].

[2] T. Schwetz, M.A. Tortola and J.W.F. Valle, *Three-flavour neutrino oscillation update*, New J. Phys. **10** (2008) 113011 [arXiv:0808.2016] [SPIRES];
G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A.M. Rotunno, *Hints of $\theta_{13} > 0$ from global neutrino data analysis*, Phys. Rev. Lett. **101** (2008) 141801 [arXiv:0806.2649] [SPIRES].

[3] G.L. Fogli, E. Lisi, A. Marrone and A. Palazzo, *Global analysis of three-flavor neutrino masses and mixings*, Prog. Part. Nucl. Phys. **57** (2006) 742 [hep-ph/0506083] [SPIRES].

[4] M. Fukugita, M. Tanimoto and T. Yanagida, *Atmospheric neutrino oscillation and a phenomenological lepton mass matrix*, Phys. Rev. **D 57** (1998) 4429 [hep-ph/9709388] [SPIRES];
M. Tanimoto, *Vacuum neutrino oscillations of solar neutrinos and lepton mass matrices*, Phys. Rev. **D 59** (1999) 017304 [hep-ph/9807283] [SPIRES]; *Large mixing angle MSW solution in S^3 flavor symmetry*, Phys. Lett. **B 483** (2000) 417 [hep-ph/0001306] [SPIRES].

[5] P.F. Harrison and W.G. Scott, *Permutation symmetry, tri-bimaximal neutrino mixing and the S^3 group characters*, Phys. Lett. **B 557** (2003) 76 [hep-ph/0302025] [SPIRES].

[6] J. Kubo, A. Mondragon, M. Mondragon and E. Rodriguez-Jauregui, *The flavor symmetry*, Prog. Theor. Phys. **109** (2003) 795 [*Erratum ibid.* **114** (2005) 287] [hep-ph/0302196] [SPIRES].

[7] S.-L. Chen, M. Frigerio and E. Ma, *Large neutrino mixing and normal mass hierarchy: a discrete understanding*, Phys. Rev. **D 70** (2004) 073008 [*Erratum ibid.* **D 70** (2004) 079905] [hep-ph/0404084] [SPIRES].

[8] W. Grimus and L. Lavoura, *$S^3 \times Z_2$ model for neutrino mass matrices*, JHEP **08** (2005) 013 [hep-ph/0504153] [SPIRES].

[9] F. Caravaglios and S. Morisi, *Neutrino masses and mixings with an S^3 family permutation symmetry*, hep-ph/0503234 [SPIRES].

[10] R.N. Mohapatra, S. Nasri and H.-B. Yu, *S^3 symmetry and tri-bimaximal mixing*, Phys. Lett. **B 639** (2006) 318 [hep-ph/0605020] [SPIRES].

[11] S. Morisi and M. Picariello, *The flavor physics in unified gauge theory from an S_3^P discrete symmetry*, Int. J. Theor. Phys. **45** (2006) 1267 [hep-ph/0505113] [SPIRES].

[12] N. Haba, A. Watanabe and K. Yoshioka, *Twisted flavors and tri/bi-maximal neutrino mixing*, Phys. Rev. Lett. **97** (2006) 041601 [hep-ph/0603116] [SPIRES].

[13] S. Kaneko, H. Sawanaka, T. Shingai, M. Tanimoto and K. Yoshioka, *Flavor symmetry and vacuum aligned mass textures*, Prog. Theor. Phys. **117** (2007) 161 [hep-ph/0609220] [SPIRES].

[14] Y. Koide, *S^3 symmetry and neutrino masses and mixings*, Eur. Phys. J. **C 50** (2007) 809 [hep-ph/0612058] [SPIRES].

[15] C.-Y. Chen and L. Wolfenstein, *Consequences of approximate S_3 symmetry of the neutrino mass matrix*, Phys. Rev. **D 77** (2008) 093009 [arXiv:0709.3767] [SPIRES].

JHEP12(2009)054

[16] F. Feruglio and Y. Lin, *Fermion mass hierarchies and flavour mixing from a minimal discrete symmetry*, *Nucl. Phys.* **B 800** (2008) 77 [arXiv:0712.1528] [SPIRES].

[17] A. Mondragon, M. Mondragon and E. Peinado, *Nearly tri-bimaximal mixing in the S_3 flavour symmetry*, *AIP Conf. Proc.* **1026** (2008) 164 [arXiv:0712.2488] [SPIRES].

[18] M. Mitra and S. Choubey, *Lepton masses in a minimal model with triplet Higgs and $S_3 \times Z_4$ flavor symmetry*, *Phys. Rev.* **D 78** (2008) 115014 [arXiv:0806.3254] [SPIRES].

[19] W. Grimus and L. Lavoura, *Tri-bimaximal lepton mixing from symmetry only*, *JHEP* **04** (2009) 013 [arXiv:0811.4766] [SPIRES].

[20] W. Grimus and L. Lavoura, *A discrete symmetry group for maximal atmospheric neutrino mixing*, *Phys. Lett.* **B 572** (2003) 189 [hep-ph/0305046] [SPIRES].

[21] W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura and M. Tanimoto, *Lepton mixing angle $\theta_{13} = 0$ with a horizontal symmetry D_4*, *JHEP* **07** (2004) 078 [hep-ph/0407112] [SPIRES];
A. Blum, R.N. Mohapatra and W. Rodejohann, *Inverted mass hierarchy from scaling in the neutrino mass matrix: low and high energy phenomenology*, *Phys. Rev.* **D 76** (2007) 053003 [arXiv:0706.3801] [SPIRES];
A. Blum, C. Hagedorn and M. Lindner, *Fermion masses and mixings from dihedral flavor symmetries with preserved subgroups*, *Phys. Rev.* **D 77** (2008) 076004 [arXiv:0709.3450] [SPIRES].

[22] W. Grimus et al., *Non-vanishing* U_{e3} *and* $\cos(2\theta_{23})$ *from a broken Z_2 symmetry*, *Nucl. Phys.* **B 713** (2005) 151 [hep-ph/0408123] [SPIRES].

[23] H. Ishimori et al., *D_4 flavor symmetry for neutrino masses and mixing*, *Phys. Lett.* **B 662** (2008) 178 [arXiv:0802.2310] [SPIRES].

[24] H. Ishimori et al., *Soft supersymmetry breaking terms from $D_4 \times Z_2$ lepton flavor symmetry*, *Phys. Rev.* **D 77** (2008) 115005 [arXiv:0803.0796] [SPIRES].

[25] Y. Kajiyama, J. Kubo and H. Okada, *D6 family symmetry and cold dark matter at LHC*, *Phys. Rev.* **D 75** (2007) 033001 [hep-ph/0610072] [SPIRES].

[26] M. Frigerio, S. Kaneko, E. Ma and M. Tanimoto, *Quaternion family symmetry of quarks and leptons*, *Phys. Rev.* **D 71** (2005) 011901 [hep-ph/0409187] [SPIRES].

[27] K.S. Babu and J. Kubo, *Dihedral families of quarks, leptons and Higgses*, *Phys. Rev.* **D 71** (2005) 056006 [hep-ph/0411226] [SPIRES];
Y. Kajiyama, E. Itou and J. Kubo, *Non-Abelian discrete family symmetry to soften the SUSY flavor problem and to suppress proton decay*, *Nucl. Phys.* **B 743** (2006) 74 [hep-ph/0511268] [SPIRES].

[28] E. Ma and G. Rajasekaran, *Softly broken A_4 symmetry for nearly degenerate neutrino masses*, *Phys. Rev.* **D 64** (2001) 113012 [hep-ph/0106291] [SPIRES];
E. Ma, *Plato's fire and the neutrino mass matrix*, *Mod. Phys. Lett.* **A 17** (2002) 2361 [hep-ph/0211393] [SPIRES].

[29] K.S. Babu, E. Ma and J.W.F. Valle, *Underlying A_4 symmetry for the neutrino mass matrix and the quark mixing matrix*, *Phys. Lett.* **B 552** (2003) 207 [hep-ph/0206292] [SPIRES].

[30] M. Hirsch, J.C. Romao, S. Skadhauge, J.W.F. Valle and A. Villanova del Moral, *Phenomenological tests of supersymmetric A_4 family symmetry model of neutrino mass*, *Phys. Rev.* **D 69** (2004) 093006 [hep-ph/0312265] [SPIRES].

[31] E. Ma, *A_4 origin of the neutrino mass matrix*, Phys. Rev. **D 70** (2004) 031901 [hep-ph/0404199] [SPIRES].

[32] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions*, Nucl. Phys. **B 720** (2005) 64 [hep-ph/0504165] [SPIRES]; *Tri-bimaximal neutrino mixing, A_4 and the modular symmetry*, Nucl. Phys. **B 741** (2006) 215 [hep-ph/0512103] [SPIRES].

[33] S.-L. Chen, M. Frigerio and E. Ma, *Hybrid seesaw neutrino masses with A_4 family symmetry*, Nucl. Phys. **B 724** (2005) 423 [hep-ph/0504181] [SPIRES].

[34] A. Zee, *Obtaining the neutrino mixing matrix with the tetrahedral group*, Phys. Lett. **B 630** (2005) 58 [hep-ph/0508278] [SPIRES].

[35] B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M.K. Parida, *A_4 symmetry and prediction of U_{e3} in a modified Altarelli-Feruglio model*, Phys. Lett. **B 638** (2006) 345 [hep-ph/0603059] [SPIRES].

[36] J.W.F. Valle, *Neutrino physics overview*, J. Phys. Conf. Ser. **53** (2006) 473 [hep-ph/0608101] [SPIRES].

[37] X.-G. He, Y.-Y. Keum and R.R. Volkas, *A_4 flavour symmetry breaking scheme for understanding quark and neutrino mixing angles*, JHEP **04** (2006) 039 [hep-ph/0601001] [SPIRES].

[38] E. Ma, H. Sawanaka and M. Tanimoto, *Quark masses and mixing with A_4 family symmetry*, Phys. Lett. **B 641** (2006) 301 [hep-ph/0606103] [SPIRES].

[39] B. Adhikary and A. Ghosal, *Constraining it CP violation in a softly broken A_4 symmetric model*, Phys. Rev. **D 75** (2007) 073020 [hep-ph/0609193] [SPIRES].

[40] E. Ma, *A_4 origin of the neutrino mass matrix*, Phys. Rev. **D 70** (2004) 031901 [hep-ph/0404199] [SPIRES]; *Aspects of the tetrahedral neutrino mass matrix*, Phys. Rev. **D 72** (2005) 037301 [hep-ph/0505209] [SPIRES]; *Supersymmetric $A_4 \times Z_3$ and A_4 realizations of neutrino tribimaximal mixing without and with corrections*, Mod. Phys. Lett. **A 22** (2007) 101 [hep-ph/0610342] [SPIRES].

[41] G. Altarelli, F. Feruglio and Y. Lin, *Tri-bimaximal neutrino mixing from orbifolding*, Nucl. Phys. **B 775** (2007) 31 [hep-ph/0610165] [SPIRES].

[42] M. Hirsch, A.S. Joshipura, S. Kaneko and J.W.F. Valle, *Predictive flavour symmetries of the neutrino mass matrix*, Phys. Rev. Lett. **99** (2007) 151802 [hep-ph/0703046] [SPIRES].

[43] S.F. King and M. Malinsky, *A_4 family symmetry and quark-lepton unification*, Phys. Lett. **B 645** (2007) 351 [hep-ph/0610250] [SPIRES].

[44] L. Lavoura and H. Kühböck, *A_4 model for the quark mass matrices*, Eur. Phys. J. **C 55** (2008) 303 [arXiv:0711.0670] [SPIRES].

[45] M. Honda and M. Tanimoto, *Deviation from tri-bimaximal neutrino mixing in A_4 flavor symmetry*, Prog. Theor. Phys. **119** (2008) 583 [arXiv:0801.0181] [SPIRES].

[46] F. Bazzocchi, S. Kaneko and S. Morisi, *A SUSY A_4 model for fermion masses and mixings*, JHEP **03** (2008) 063 [arXiv:0707.3032] [SPIRES].

[47] F. Bazzocchi, M. Frigerio and S. Morisi, *Fermion masses and mixing in models with* $SO(10) \times A_4$ *symmetry*, Phys. Rev. **D 78** (2008) 116018 [arXiv:0809.3573] [SPIRES].

[48] Y. Lin, *A predictive A_4 model, charged lepton hierarchy and tri-bimaximal sum rule*, Nucl. Phys. **B 813** (2009) 91 [arXiv:0804.2867] [SPIRES].

[49] A. Hayakawa, H. Ishimori, Y. Shimizu and M. Tanimoto, *Deviation from tri-bimaximal mixing and flavor symmetry breaking in a seesaw type A_4 model*, Phys. Lett. **B 680** (2009) 334 [arXiv:0904.3820] [SPIRES].

[50] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, *Lepton flavour violation in models with A_4 flavour symmetry*, Nucl. Phys. **B 809** (2009) 218 [arXiv:0807.3160] [SPIRES].

[51] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, *Tri-bimaximal neutrino mixing and quark masses from a discrete flavour symmetry*, Nucl. Phys. **B 775** (2007) 120 [hep-ph/0702194] [SPIRES].

[52] M.-C. Chen and K.T. Mahanthappa, *CKM and tri-bimaximal MNS matrices in a* $\mathrm{SU}(5)\times^{(d)} T$ *model*, Phys. Lett. **B 652** (2007) 34 [arXiv:0705.0714] [SPIRES].

[53] A. Aranda, *Neutrino mixing from the double tetrahedral group T'*, Phys. Rev. **D 76** (2007) 111301 [arXiv:0707.3661] [SPIRES].

[54] D.A. Eby, P.H. Frampton and S. Matsuzaki, *Predictions of neutrino mixing angles in a T' model*, Phys. Lett. **B 671** (2009) 386 [arXiv:0810.4899] [SPIRES].

[55] P.H. Frampton and T.W. Kephart, *Simple non-Abelian finite flavor groups and fermion masses*, Int. J. Mod. Phys. **A 10** (1995) 4689 [hep-ph/9409330] [SPIRES].

[56] G.-J. Ding, *Fermion mass hierarchies and flavor mixing from T' symmetry*, Phys. Rev. **D 78** (2008) 036011 [arXiv:0803.2278] [SPIRES].

[57] E. Ma, *Neutrino mass matrix from S^4 symmetry*, Phys. Lett. **B 632** (2006) 352 [hep-ph/0508231] [SPIRES].

[58] C. Hagedorn, M. Lindner and R.N. Mohapatra, *S^4 flavor symmetry and fermion masses: towards a grand unified theory of flavor*, JHEP **06** (2006) 042 [hep-ph/0602244] [SPIRES].

[59] Y. Cai and H.-B. Yu, *An* SO(10) *GUT model with S4 flavor symmetry*, Phys. Rev. **D 74** (2006) 115005 [hep-ph/0608022] [SPIRES].

[60] H. Zhang, *Flavor $S^4 \times Z_2$ symmetry and neutrino mixing*, Phys. Lett. **B 655** (2007) 132 [hep-ph/0612214] [SPIRES].

[61] Y. Koide, *S_4 flavor symmetry embedded into* SU(3) *and lepton masses and mixing*, JHEP **08** (2007) 086 [arXiv:0705.2275] [SPIRES].

[62] F. Bazzocchi and S. Morisi, *S4 as a natural flavor symmetry for lepton mixing*, Phys. Rev. **D 80** (2009) 096005 [arXiv:0811.0345] [SPIRES].

[63] H. Ishimori, Y. Shimizu and M. Tanimoto, *S4 flavor symmetry of quarks and leptons in* SU(5) *GUT*, Prog. Theor. Phys. **121** (2009) 769 [arXiv:0812.5031] [SPIRES].

[64] F. Bazzocchi, L. Merlo and S. Morisi, *Fermion masses and mixings in a S4-based model*, Nucl. Phys. **B 816** (2009) 204 [arXiv:0901.2086] [SPIRES].

[65] F. Bazzocchi, L. Merlo and S. Morisi, *Phenomenological consequences of see-saw in S4 based models*, Phys. Rev. **D 80** (2009) 053003 [arXiv:0902.2849] [SPIRES].

[66] G.C. Branco, J.M. Gerard and W. Grimus, *Geometrical T violation*, Phys. Lett. **B 136** (1984) 383 [SPIRES].

[67] C. Luhn, S. Nasri and P. Ramond, *The flavor group* $\Delta(3n^2)$, *J. Math. Phys.* **48** (2007) 073501 [hep-th/0701188] [SPIRES].

[68] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-Abelian discrete family symmetry*, *Phys. Lett.* **B 648** (2007) 201 [hep-ph/0607045] [SPIRES].

[69] E. Ma, *Neutrino mass matrix from* Δ_{27} *symmetry*, *Mod. Phys. Lett.* **A 21** (2006) 1917 [hep-ph/0607056] [SPIRES].

[70] E. Ma, *Near tribimaximal neutrino mixing with* $\Delta(27)$ *symmetry*, *Phys. Lett.* **B 660** (2008) 505 [arXiv:0709.0507] [SPIRES].

[71] T. Kobayashi, J. Kubo and H. Terao, *Exact* S^3 *symmetry solving the supersymmetric flavor problem*, *Phys. Lett.* **B 568** (2003) 83 [hep-ph/0303084] [SPIRES].

[72] P. Ko, T. Kobayashi, J.-H. Park and S. Raby, *String-derived* D_4 *flavor symmetry and phenomenological implications*, *Phys. Rev.* **D 76** (2007) 035005 [*Erratum ibid.* **D 76** (2007) 059901] [arXiv:0704.2807] [SPIRES].

[73] H. Ishimori, T. Kobayashi, Y. Omura and M. Tanimoto, *Soft supersymmetry breaking terms from A4 lepton flavor symmetry*, *JHEP* **12** (2008) 082 [arXiv:0807.4625] [SPIRES].

[74] O.M. Vives, *Flavour symmetries and the SUSY soft-breaking sector at the LHC*, talk at the 16^{th} *YKIS*, Kyoto Japan January 28 2009.

[75] T. Kobayashi, S. Raby and R.-J. Zhang, *Searching for realistic 4D string models with a Pati-Salam symmetry: orbifold grand unified theories from heterotic string compactification on a* Z_6 *orbifold*, *Nucl. Phys.* **B 704** (2005) 3 [hep-ph/0409098] [SPIRES].

[76] T. Kobayashi, H.P. Nilles, F. Ploger, S. Raby and M. Ratz, *Stringy origin of non-Abelian discrete flavor symmetries*, *Nucl. Phys.* **B 768** (2007) 135 [hep-ph/0611020] [SPIRES].

[77] H. Abe, K.-S. Choi, T. Kobayashi and H. Ohki, *Non-Abelian discrete flavor symmetries from magnetized/intersecting brane models*, *Nucl. Phys.* **B 820** (2009) 317 [arXiv:0904.2631] [SPIRES].

[78] C.S. Lam, *The unique horizontal symmetry of leptons*, *Phys. Rev.* **D 78** (2008) 073015 [arXiv:0809.1185] [SPIRES].

[79] J.A. Escobar and C. Luhn, *The flavor group* $\Delta(6n^2)$, *J. Math. Phys.* **50** (2009) 013524 [arXiv:0809.0639] [SPIRES].

[80] H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu and M. Tanimoto, *Lepton flavor model from Delta*(54) *symmetry*, *JHEP* **04** (2009) 011 [arXiv:0811.4683] [SPIRES].

[81] T. Kobayashi, Y. Omura and K. Yoshioka, *Flavor symmetry breaking and vacuum alignment on orbifolds*, *Phys. Rev.* **D 78** (2008) 115006 [arXiv:0809.3064] [SPIRES].

[82] G. Seidl, *Unified model of fermion masses with Wilson line flavor symmetry breaking*, arXiv:0811.3775 [SPIRES].

[83] V.S. Kaplunovsky and J. Louis, *Model independent analysis of soft terms in effective supergravity and in string theory*, *Phys. Lett.* **B 306** (1993) 269 [hep-th/9303040] [SPIRES].

[84] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of FCNC and CP constraints in general SUSY extensions of the Standard Model*, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387] [SPIRES].

[85] J.M. Frere, D.R.T. Jones and S. Raby, *Fermion masses and induction of the weak scale by supergravity*, *Nucl. Phys.* **B 222** (1983) 11 [SPIRES];
L. Álvarez-Gaumé, J. Polchinski and M.B. Wise, *Minimal low-energy supergravity*, *Nucl. Phys.* **B 221** (1983) 495 [SPIRES];
C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quirós, *Low-energy behavior of realistic locally supersymmetric Grand Unified Theories*, *Nucl. Phys.* **B 236** (1984) 438 [SPIRES];
M. Claudson, L.J. Hall and I. Hinchliffe, *Low-energy supergravity: false vacua and vacuous predictions*, *Nucl. Phys.* **B 228** (1983) 501 [SPIRES];
J.A. Casas, A. Lleyda and C. Muñoz, *Strong constraints on the parameter space of the MSSM from charge and color breaking minima*, *Nucl. Phys.* **B 471** (1996) 3 [hep-ph/9507294] [SPIRES].

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